



A Novel Multi-Criteria Decision-Making Approach for the Solution of Cigarette Use Reduction Based on Generalized Neutrosophic Hypersoft Sets

Nguyen Thu Huong¹, Nguyen Tho Thong^{2,*}, Dao The Son³, Dao Cao Son¹ and Florentin Smarandache⁴

¹Faculty of Marketing, Thuongmai University, Ho Tung Mau, Cau Giay, Hanoi, 010000, Vietnam;

huongnt.t@tmu.edu.vn, son.dc@tmu.edu.vn

²Faculty of Computer Science and Engineering, Thuyloi University, 175 Tay Son, Dong Da, Hanoi, 010000, Vietnam; thongnt89@tlu.edu.vn

³Center for Economics and Community Development, Ngoc Ha, Doi Can, Ba Dinh, Hanoi, 010000, Vietnam; daotheron@gmail.com

⁴Department of Mathematics, University of New Mexico, 705 Gurley Ave., Gallup, New Mexico 87301, State, United States; fsmarandache@gmail.com

*Correspondence: thongnt89@tlu.edu.vn;

Abstract. Nowadays, decision-making becomes an intricate matter due to an ambiguous, imprecise, and indeterminate setting, particularly in cases with multiple attributes that consider the relationships among sub-attributes and the availability of decision-makers. The deficiency of suitable skills and constrained models has resulted in significant mistakes in decision-making within this context. This study proposes a generalized neutrosophic hypersoft set (GNHSS) to solve uncertainty, indeterminacy, inconsistency, and the relationships between sub-attributes through hesitation of estimated values in decision-making issues. Along with introducing the new concept of GNHSS, this study also presents some operations and measures to complete the mathematical space of the proposed theory. Then, an advanced model derived from the Technique for Order Preference by Similarity to Ideal Solution, which is based on GNHSS (GNHSS-TOPSIS), is presented to solve challenges related to uncertainty, indeterminacy, inconsistency, and relationships between sub-attributes, as well as hesitation in estimated values in complex decision-making problems. Furthermore, the feasibility and soundness of addressing decision-making problems through the proposed model are demonstrated by an example in curious topic in the real world that is developing a solution for cigarette use reduction.

Keywords: Neutrosophic HyperSoft Set; Neutrosophic set; Hesitant Fuzzy Sets; MCDM;

1. Introduction

Multi-criteria decision-making (MCDM) is a helpful strategy that considers numerous factors simultaneously while making decisions. MCDM has proven to be a successful tool utilized in real-world scenarios across domains like social media [1, 2, 3], economics [4, 5], and medicine [6, 7], etc. Nowadays, decision-making is often a complex process, and decision-makers frequently rely on a combination of quantitative data and qualitative insights driven by human intuition. Therefore, harnessing these intuitive data effectively requires robust tools that can handle the inherent uncertainty, indeterminacy, and imprecision; as well as datasets with multiple attribute-valued disjoint sets in decision-making.

Numerous studies have been suggested to tackle the challenges encountered in decision-making. For example, Zadeh [8] proposed the idea of fuzzy sets (FS) in 1965, a groundbreaking contribution to dealing with uncertainty and vagueness in various fields of real life. The success of FS led to the expansion of numerous studies focusing on FS theory to tackle real-world issues. Atanassov et al. [9] introduced intuitionistic fuzzy sets (IFS). IFS differs from FS by introducing the concept of a non-membership degree, which quantifies the extent to which an element does not belong to a set. This complements the degree of membership, which quantifies the degree of belongingness. IFS provides a broader framework for managing information that is fuzzy, uncertain, or incomplete. Torra [10] presented the idea of hesitant fuzzy sets. This concept combines the notions of fuzzy sets and hesitant behavior, providing a framework to handle imprecise and uncertain information as a solution to lattice compatibility issues in fuzzy sets that often arise in group decision-making scenarios. These sets allow decision-makers to express hesitation, enhancing the reliability of results when assigning membership values to elements, whether in quantitative or qualitative settings in an uncertain multi-attribute decision-making context. The efficacy and successful implementation of the HFS theory have been validated in numerous published studies [11, 12, 13, 14, 15].

Although FS theory or their extended versions can cope with ambiguous data, they have difficulties when attempting to model incomplete data. Smarandache [16] introduced the concept of the Neutrosophic set, which is a scientific tool for solving problems such as ambiguous, indeterminate, and contradictory information. The Neutrosophic set displays membership values for truth, indeterminacy, and falsity. This concept is useful in a variety of applications due to its ability to effectively handle indeterminacy. Indeterminacy has been thoroughly tested [17, 18, 19]. Furthermore, the membership values of truth, indeterminacy, and falsehood are independent. Exploring the realm of human intention involves delving into theoretical studies that contemplate single factors such as uncertainty, indeterminacy, and inconsistency, along with their various extensions. Molodtsov [20] proposed the theory of soft sets, offering a systematic approach to addressing uncertainty by utilizing soft sets, defined by multivalued parameter

functions under certain conditions. Next, in 2018, Smarandache [21] proposed A Hypersoft set (HSS) theory that is an extension of soft set theory. HSS is an advanced mathematical concept that extends traditional soft set theory to handle data with multiple attribute-valued disjoint sets, offering a more comprehensive approach to dealing with uncertainties and complex decision-making processes. Hypersoft sets have found applications in decision-making problems, graph theory [22], game theory [23], operation research [24], and more, offering a way to address real-world challenges where data ambiguity is prevalent. Researchers have continuously extended the theory by combining it with traditional theories. Various models, such as, the q-rung orthopair fuzzy hypersoft sets (q-ROFHSSs) [25] and Pythagorean fuzzy hypersoft sets (PFHSS) [26] have been developed to enhance decision-making accuracy and efficiency by introducing innovative aggregation operators based on mathematical principles like Einstein's operational laws. Numerous studies have also incorporated the hypersoft set and neutrosophic theory, thus completing the mathematical space, and providing a more robust framework for real-world problem-solving and decision-making scenarios [27].

The concept of the neutrosophic hypersoft set was first considered by Smarandache in [21]. Future operations, measures, or theoretical extensions for the mathematical space of NHSS and corresponding decision-making methods are also proposed. In the subsequent research conducted by Saqlain and colleagues regarding the Neutrosophic hypersoft set, the authors [28] extended the TOPSIS method for neutrosophic hypersoft set-based issues, addressing linguistic assessments using the accuracy function. The suggested method is simple to build, precise, and useful for addressing MCDM issues using multiple-valued neutrosophic data. Between 2020 and 2023, the authors in [24, 27, 29, 30] also proposed basic operators such as union, intersection, complement, subset, null set, and equal set; and the similarity measure of single-valued neutrosophic hypersoft sets. They also considered distances, similarity measures, and aggregated operations for aggregating the NHSS decision matrix. Furthermore, the authors combined multi-polar interval-valued neutrosophic sets with hypersoft sets. Similarity measures were established for mPIVNHSSs, and an algorithm was introduced for everyday life problems. K-Nearest Neighbor (KNN) is used for ranking sites, mimicking human behavior for personalized suggestions. Jafar et al. [31] merged neutrosophic and hypersoft set theories to propose new distances and max-min operators. The study compares NHSS with existing methodologies, demonstrating its superiority in decision-making with more bifurcated attributes. The proposed work can be applied to hypersoft set hybrids like Fuzzy, Intuitionistic, Bipolar, m-polar, and Pythagorean. the authors [32] also proposed trigonometric similarity measures for Neutrosophic HyperSoft Sets (NHSSs), which involve cosine and cotangent measures. The metrics are used for renewable energy source selection challenges, showing the optimal geographical location for energy production units. Rahman et al. [33] introduced

an innovative methodology known as fuzzy parameterized possibility single valued neutrosophic hypersoft set (Fppsv-NHSS). The algorithm integrates Sanchez's method to resolve the SOWSSP, addressing uncertainties in decision-makers' choices of parameters and approximations. The algorithm manages uncertainties by utilizing fuzzy parameterization and possibility grades, allowing decision-makers to evaluate alternatives independently. Dhumras et al. [34] presented a new Hellinger information measure for a single-valued neutrosophic hypersoft set, which addresses vagueness and impreciseness in decision-making models. Zhao et al. [35] proposed a neutrosophic hypersoft set hybrid known as the potential single-valued neutrosophic hypersoft set (psv-NHSS) for evaluating investment projects through its aggregation operations and decision-support system. The former describes the innovative concept of psv-NHSS and its aggregations. The latter created a decision-support system using aggregations such as core matrix, maximum-valued decision, minimum-valued decision, and scoring-valued decision from psv-NHSS. The adopted method was applied to real-world scenarios to evaluate hydroelectric power station projects for investment purposes. Zulqarnain et al. [36] introduced aggregation operators (AOs) for NHSS, including the neutrosophic hypersoft weighted average (NHSWA) and neutrosophic hypersoft weighted geometric (NHSWG) operators, as well as their required features. They developed a multi-criteria decision-making approach for solid waste management site selection. Ihsan et al. [37] pointed out that the fuzzy parameterized neutrosophic hypersoft expert set (FpNHse-set) is a valuable extension of the neutrosophic soft expert set and hypersoft set. It uses a multi-argument approximate function to classify characters into sub-characteristic valued sets, making decision-making more adaptable and dependable. They used FpNHse-sets and Sanchez's method for medical diagnosis of heart disease, ensuring its accuracy through comparisons with existing studies. Additionally, a significant number of scholarly research efforts have implemented NHSS theory in practical decision-making situations, such as Angelica et al. [38] designed a neutrosophic hypersoftset model to measure the tax culture of Ecuadorian Small and Medium Enterprises in Ecuador. Ajabnoor et al. [39] expanded The Multi Attributive Border Approximation Area Comparison (MABAC) model in the Neutrosophic HyperSoft context and applied the model to evaluate distributed leadership in education. Saeed et al. [40] demonstrated a decision-making methodology grounded in Pythagorean Neutrosophic Hypersoft Sets pertaining to the issue of work-life balance.

Motivation

(1) The survey above reveals that NHSS theory has captured the interest of numerous researchers and demonstrates its effectiveness and adaptability in tackling complex practical issues, such as dealing with relationships between sub-attributes and various forms of uncertainty, indeterminacy, and inconsistency in decision-making problems.

(2) Nevertheless, when facing numerous practical decision-making scenarios containing sub-attributes and their relationships, experts may lack full preparedness to reach conclusions or may exhibit hesitance during the estimation phase.

(3) The issue remains inadequately addressed in the realm of traditional NHSS theory in published studies, with only minimal acknowledgment. The primary driving force behind this paper lies in the deficiencies observed in existing theories and decision-support methodologies.

Contributions

In this study, we introduce GNHSS theory along with its operations and measures. Thereby introducing a decision-making method based on the proposed theory to solve complex multi-criteria decision-making problems in real-life. The contributions and novelty of this paper are specifically emphasized in the following manners:

(1) Propose a generalized neutrosophic hypersoft set based on the combination of NHSS theory and a hesitant fuzzy set to address the problems related to uncertainty, indeterminacy, inconsistency, and sub-attribute relationship representation, as well as hesitation in estimated values.

(2) Along with the new framework of GNHSS, this study introduces some operations and measures to complete the mathematical framework of the proposed theory.

(3) Develop the GNHSS-TOPSIS decision-making model to solve decision-making for problems.

(4) Demonstrate the feasibility and soundness of applying the suggested model to decision-making situations. This is illustrated with a real-world example involving the solution of cigarette use reduction, a long-standing societal concern.

The framework of this work is organized as follows: Section 2 introduces some fundamental principles. Section 3 presents the novel idea of generalized neutrosophic hypersoft sets, operators, and their measures. Section 4 provides a new decision-making model based on GNHSS theory. The application of the proposed model involved a solution for reducing cigarette use. In Section 6, the paper concludes with a summary and discussion of the findings.

2. Preliminaries

Definition 2.1 ([41]). *Let U be the universal set and $P(U)$ be the power set of U . Consider $n \geq 1$ and $AT_1, AT_2, \dots, AT_n$ as well-defined attributes, whose corresponding attributive values are respectively the set $A_1, A_2, \dots, A_n$ with $A^i \cap A^j$ for $i \neq j$ and $i, j \in \{1, 2, \dots, n\}$ and their relation $A_1 \times A_2 \times A_3 \times \dots \times A_n = \Lambda$, then pair (Γ, Λ) referred to as Neutrosophic Hypersoft set (NHSS) over U where,*

$$\begin{aligned}\Gamma : A_1 \times A_2 \times A_3 \times \dots \times A_n &\rightarrow P(U); \\ \Gamma(A_1 \times A_2 \times A_3 \times \dots \times A_n) &= \{\langle x, T(\Gamma(\Lambda)), I(\Gamma(\Lambda)), F(\Gamma(\Lambda)) \rangle, x \in U\};\end{aligned}\quad (1)$$

Where T is the membership value of truthiness, I is the membership value of indeterminacy and F is the membership value of falsity such that $T, I, F : U \rightarrow [0, 1]$ also $0 \leq T(\Gamma(\Lambda)) + I(\Gamma(\Lambda)) + F(\Gamma(\Lambda)) \leq 3$.

Definition 2.2 ([10]). Let U be a nonempty set, a hesitant fuzzy set E on U is defined in terms of a function $h_e(x)$ such that, when applied to U , it returns a finite subset of $[0, 1]$, which can be represented as the following mathematical symbol:

$$h_E(x) = \{\langle x, h_E(x) \rangle | x \in U\} \quad (2)$$

Where $h_E(x)$ is a set of some different values in $[0, 1]$.

3. Generalized Neutrosophic Hypersoft Sets

Extending NHS with the notion of HFS, we introduce the concepts of Generalized Generalized Neutrosophic Hypersoft set (GNHS) and Generalized Neutrosophic Hypersoft element (GNHE), which include basic elements, operational laws and the score functions.

Definition 3.1. Let U be the universal set and $P(U)$ be the power set of U . Consider $n \geq 1$ and $AT_1, AT_2, \dots, AT_n$ as well-defined attributes, whose corresponding attributive values are respectively the set $\{A_1, A_2, A_3, \dots, A_n\}$ with $A_i \cap A_j = \emptyset$ for $i \neq j$ and $i, j \in \{1, 2, 3, \dots, n\}$ and $\Lambda = A_1 \times A_2 \times A_3 \times \dots \times A_n; n \geq 1$ denotes their relation, then pair (Γ, Λ) referred to as Generalized Neutrosophic Hypersoft set (GNHSS) over U where Γ is a mapping,

$$\begin{aligned}\Gamma : \Lambda = A_1 \times A_2 \times A_3 \times \dots \times A_n &\rightarrow P(U) \\ \Gamma(\Lambda) &= \left\{ u, \langle \tilde{T}(u, \Gamma(\Lambda)), \tilde{I}(u, \Gamma(\Lambda)), \tilde{F}(u, \Gamma(\Lambda)) \rangle \right\}\end{aligned}\quad (3)$$

Where $\tilde{T}(u, \Gamma(\Lambda)), \tilde{I}(u, \Gamma(\Lambda)), \tilde{F}(u, \Gamma(\Lambda))$ denote the possible truth hesitant membership degree, indeterminacy hesitant membership degree and the falsity hesitant membership degree of the element $u \in U$; and $\tilde{T}(u, \Gamma(\Lambda)), \tilde{I}(u, \Gamma(\Lambda)), \tilde{F}(u, \Gamma(\Lambda))$ is a set of some different values in $[0, 1]$;

also $0 \leq \tilde{T}(u, \Gamma(\Lambda))^+ + \tilde{I}(u, \Gamma(\Lambda))^+ + \tilde{F}(u, \Gamma(\Lambda))^+ \leq 3$ with $\tilde{T}(u, \Gamma(\Lambda))^+ = \max(\tilde{T}(u, \Gamma(\Lambda)))$; $\tilde{I}(u, \Gamma(\Lambda))^+ = \max(\tilde{I}(u, \Gamma(\Lambda)))$; $\tilde{F}(u, \Gamma(\Lambda))^+ = \max(\tilde{F}(u, \Gamma(\Lambda)))$;

Simply a Generalized Neutrosophic Hypersoft element (GNHSE) of $u \in U$ can be stated as $\Gamma(\Lambda) = \langle \tilde{T}(\Gamma(\Lambda)), \tilde{I}(\Gamma(\Lambda)), \tilde{F}(\Gamma(\Lambda)) \rangle$;

For an n-tuple $\hat{\gamma}_j = (\gamma_{1j}, \gamma_{2j}, \gamma_{3j}, \dots, \gamma_{nj})$. A Generalized Neutrosophic Hypersoft Number (GNHSSN) of $u_i \in U$ and $\hat{\gamma}_j \in \Lambda$ is also defined as $\Gamma(\Lambda)_i(\hat{\gamma}_j) = \langle \tilde{T}(u_i, \hat{\gamma}_j), \tilde{I}(u_i, \hat{\gamma}_j), \tilde{F}(u_i, \hat{\gamma}_j) \rangle$. For simplicity, we can written $\langle \tilde{T}(\Gamma_{ij}(\Lambda)), \tilde{I}(\Gamma_{ij}(\Lambda)), \tilde{F}(\Gamma_{ij}(\Lambda)) \rangle$

Example 3.1. Assume that a person is evaluating the attractiveness of a living space. Let U be a universe consisting of two choices $U = \{u_1, u_2\}$ and $\Lambda = \{A_1, A_2\}$ and $A_1 = \{A_{11}, A_{12}\}$ and $A_2 = \{A_{21}, A_{22}\}$. Let $\Lambda = A_1 \times A_2$ be a set attributes,

$$\begin{aligned}\Lambda &= A_1 \times A_2 = \{A_{11}, A_{12}\} \\ &= \{(A_{11}, A_{21}), (A_{11}, A_{22}), (A_{12}, A_{21}), (A_{12}, A_{22})\}; \\ \Lambda &= \{\hat{\gamma}_1, \hat{\gamma}_2, \hat{\gamma}_3, \hat{\gamma}_4\}\end{aligned}$$

Then, the GNHSS is given as

$$(\Gamma, \Lambda) = \left\{ \begin{aligned} &\langle u_1, (\hat{\gamma}_1 \langle \{0.1, 0.5\}, \{0.3, 0.6\}, \{0.4, 0.5\} \rangle, \hat{\gamma}_2 \langle \{0.8\}, \{0.2, 0.6\}, \{0.1, 0.3, 0.7\} \rangle) \rangle \\ &\langle u_2, (\hat{\gamma}_3 \langle \{0.3, 0.7\}, \{0.1, 0.5\}, \{0.5, 0.8\} \rangle, \hat{\gamma}_4 \langle \{0.5, 0.8\}, \{0.3, 0.8\}, \{0.2, 0.6\} \rangle) \rangle \end{aligned} \right\}$$

Definition 3.2. Let $\Gamma(\Lambda)$ be a GNHSS over U ; consider $l_1, l_2, l_3, \dots, l_n$ for $n \geq 1$, be n well-defined attributes, whose corresponding attributive values are respectively the set $A_1, A_2, A_3, \dots, A_n$ with $A_i \cap A_j = \emptyset$ for $i \neq j$ and $i, j \in \{1, 2, 3, \dots, n\}$ and their relation $\Lambda = A_1 \times A_2 \times A_3 \times \dots \times A_n; n \geq 1$ then $\Gamma(\Lambda)$ is Empty or Null Generalized Neutrosophic Hypersoft set if

$$\bigcup_{\sigma \in \tilde{T}(\Gamma(\Lambda))} \sigma = 0; \quad \bigcup_{\nu \in \tilde{T}(\Gamma(\Lambda))} \nu = 0; \quad \bigcup_{\eta \in \tilde{T}(\Gamma(\Lambda))} \eta = 0; \quad (4)$$

Example 3.2. Considering Example 3.1,

$$(\Gamma, \Lambda) = \left\{ \begin{aligned} &\langle u_1, (\hat{\gamma}_1 \langle \{0, 0\}, \{0, 0\}, \{0, 0\} \rangle, \hat{\gamma}_2 \langle \{0\}, \{0, 0\}, \{0, 0, 0\} \rangle) \rangle \\ &\langle u_2, (\hat{\gamma}_3 \langle \{0, 0\}, \{0, 0\}, \{0, 0\} \rangle, \hat{\gamma}_4 \langle \{0, 0\}, \{0, 0\}, \{0, 0\} \rangle) \rangle \end{aligned} \right\}$$

Definition 3.3. Let $\Gamma(\Lambda)$, $\Gamma(\Lambda_1)$ and $\Gamma(\Lambda_2)$ be three GNHSS over U ; Operators of GNHSS as following

Complement: Complement of $\Gamma(\Lambda)$ is written as $\Gamma^c(\Lambda) = \langle \tilde{T}^c(\Gamma(\Lambda)), \tilde{I}^c(\Gamma(\Lambda)), \tilde{F}^c(\Gamma(\Lambda)) \rangle$ and defined as,

$$\tilde{T}^c(\Gamma(\Lambda)) = \tilde{F}(\Gamma(\Lambda)); \tilde{I}^c(\Gamma(\Lambda)) = \bigcup_{\nu \in \tilde{T}(\Gamma(\Lambda))} 1 - \nu; \tilde{F}^c(\Gamma(\Lambda)) = \tilde{T}(\Gamma(\Lambda)); \quad (5)$$

Example 3.3. Considering Example 3.1,

$$(\Gamma, \Lambda) = \left\{ \langle u_1, (\hat{\gamma}_1 \langle \{0.4, 0.5\}, \{0.7, 0.4\}, \{0.1, 0.5\}\rangle, \hat{\gamma}_2 \langle \{0.1, 0.3, 0.7\}, \{0.8, 0.4\}, \{0.8\}\rangle) \rangle \right. \\ \left. \langle u_2, (\hat{\gamma}_3 \langle \{0.5, 0.8\}, \{0.9, 0.5\}, \{0.3, 0.7\}\rangle, \hat{\gamma}_4 \langle \{0.2, 0.6\}, \{0.7, 0.5\}, \{0.5, 0.8\}\rangle) \rangle \right\}$$

Union of two Generalized Neutrosophic Hypersoft set

$$\Gamma(\Lambda_1) \cup \Gamma(\Lambda_2) = \langle \tilde{T}_{\Gamma(\Lambda_1) \cup \Gamma(\Lambda_2)}, \tilde{I}_{\Gamma(\Lambda_1) \cup \Gamma(\Lambda_2)}, \tilde{F}_{\Gamma(\Lambda_1) \cup \Gamma(\Lambda_2)} \rangle \quad (6)$$

Where

$$\tilde{T}_{\Gamma(\Lambda_1) \cup \Gamma(\Lambda_2)} = \begin{cases} \tilde{T}_{\Gamma(\Lambda_1)} & \text{if } \gamma \in \Lambda_1 \setminus \Lambda_2 \\ \tilde{T}_{\Gamma(\Lambda_2)} & \text{if } \gamma \in \Lambda_2 \setminus \Lambda_1 \\ \bigcup_{\sigma_1 \in \tilde{T}_{\Gamma(\Lambda_1)}, \sigma_2 \in \tilde{T}_{\Gamma(\Lambda_2)}} \hat{\sigma}_{12} \geq \max(\min(\tilde{T}_{\Gamma(\Lambda_1)}), \min(\tilde{T}_{\Gamma(\Lambda_2)})) & \text{if } \gamma \in \Lambda_2 \cap \Lambda_1 \end{cases} \quad (7)$$

$$\tilde{I}_{\Gamma(\Lambda_1) \cup \Gamma(\Lambda_2)} = \begin{cases} \tilde{I}_{\Gamma(\Lambda_1)} & \text{if } \gamma \in \Lambda_1 \setminus \Lambda_2 \\ \tilde{I}_{\Gamma(\Lambda_2)} & \text{if } \gamma \in \Lambda_2 \setminus \Lambda_1 \\ \bigcup_{\nu_1 \in \tilde{I}_{\Gamma(\Lambda_1)}, \nu_2 \in \tilde{I}_{\Gamma(\Lambda_2)}} \hat{\nu}_{12} \geq \min(\max(\tilde{I}_{\Gamma(\Lambda_1)}), \max(\tilde{I}_{\Gamma(\Lambda_2)})) & \text{if } \gamma \in \Lambda_2 \cap \Lambda_1 \end{cases} \quad (8)$$

$$\tilde{F}_{\Gamma(\Lambda_1) \cup \Gamma(\Lambda_2)} = \begin{cases} \tilde{F}_{\Gamma(\Lambda_1)} & \text{if } \gamma \in \Lambda_1 \setminus \Lambda_2 \\ \tilde{F}_{\Gamma(\Lambda_2)} & \text{if } \gamma \in \Lambda_2 \setminus \Lambda_1 \\ \bigcup_{\eta_1 \in \tilde{F}_{\Gamma(\Lambda_1)}, \eta_2 \in \tilde{F}_{\Gamma(\Lambda_2)}} \hat{\eta}_{12} \geq \min(\max(\tilde{F}_{\Gamma(\Lambda_1)}), \max(\tilde{F}_{\Gamma(\Lambda_2)})) & \text{if } \gamma \in \Lambda_2 \cap \Lambda_1 \end{cases} \quad (9)$$

Example 3.4. Let two GNHSS $\Gamma(\Lambda_1)$ and $\Gamma(\Lambda_2)$ as below

$$(\Gamma, \Lambda_1) = \left\{ \langle u_1, \left(\hat{\gamma}_1, \langle \{0.1, 0.5\}, \{0.3, 0.6\}, \{0.4, 0.5\} \rangle, \hat{\gamma}_2, \langle \{0.8\}, \{0.2, 0.6\}, \{0.1, 0.3, 0.7\} \rangle \right) \rangle \right. \\ \left. \langle u_2, \left(\hat{\gamma}_3, \langle \{0.3, 0.7\}, \{0.1, 0.5\}, \{0.5, 0.8\} \rangle, \hat{\gamma}_4, \langle \{0.6, 0.8\}, \{0.3, 0.5\}, \{0.2, 0.6\} \rangle \right) \rangle \right\} \\ (\Gamma, \Lambda_2) = \left\{ \langle u_1, \left(\hat{\gamma}_1, \langle \{0.4\}, \{0.2, 0.3\}, \{0.6\} \rangle, \hat{\gamma}_2, \langle \{0.4, 0.6\}, \{0.3\}, \{0.2\} \rangle \right) \rangle \right. \\ \left. \langle u_2, \left(\hat{\gamma}_3, \langle \{0.2, 0.5\}, \{0.4, 0.7\}, \{0.6\} \rangle, \hat{\gamma}_4, \langle \{0.7\}, \{0.5, 0.6\}, \{0.1, 0.3\} \rangle \right) \rangle \right\}$$

$$(\Gamma, \Lambda_1) \cup (\Gamma, \Lambda_2) = \left\{ \begin{array}{l} \langle u_1, \left(\hat{\gamma}_1, \langle \{0.4, 0.5\}, \{0.5, 0.6\}, \{0.5, 0.6\} \rangle, \hat{\gamma}_2, \langle \{0.8\}, \{0.5, 0.6\}, \{0.4, 0.7\} \rangle \right) \\ \langle u_2, \left(\hat{\gamma}_1, \langle \{0.4\}, \{0.2, 0.3\}, \{0.6\} \rangle, \hat{\gamma}_2, \langle \{0.4, 0.6\}, \{0.3\}, \{0.2\} \rangle \right) \\ \langle u_3, \left(\hat{\gamma}_1, \langle \{0.6, 0.8\}, \{0.8, 0.9\}, \{0.3, 0.7\} \rangle, \hat{\gamma}_2, \langle \{0.8\}, \{0.1, 0.4\}, \{0.2\} \rangle \right) \end{array} \right\}$$

Intersection of two Generalized Neutrosophic Hypersoft set

$$\Gamma(\Lambda_1) \cap \Gamma(\Lambda_2) = \langle \tilde{T}_{\Gamma(\Lambda_1) \cap \Gamma(\Lambda_2)}, \tilde{I}_{\Gamma(\Lambda_1) \cap \Gamma(\Lambda_2)}, \tilde{F}_{\Gamma(\Lambda_1) \cap \Gamma(\Lambda_2)} \rangle \quad (10)$$

Where

$$\tilde{T}_{\Gamma(\Lambda_1) \cap \Gamma(\Lambda_2)} = \begin{cases} \tilde{T}_{\Gamma(\Lambda_1)} & \text{if } \gamma \in \Lambda_1 \setminus \Lambda_2 \\ \tilde{T}_{\Gamma(\Lambda_2)} & \text{if } \gamma \in \Lambda_2 \setminus \Lambda_1 \\ \bigcup_{\sigma_1 \in \tilde{T}_{\Gamma(\Lambda_1)}, \sigma_2 \in \tilde{T}_{\Gamma(\Lambda_2)}} \hat{\sigma}_{12} \geq \min(\max(\tilde{T}_{\Gamma(\Lambda_1)}), \max(\tilde{T}_{\Gamma(\Lambda_2)})) & \text{if } \gamma \in \Lambda_2 \cap \Lambda_1 \end{cases} \quad (11)$$

$$\tilde{I}_{\Gamma(\Lambda_1) \cap \Gamma(\Lambda_2)} = \begin{cases} \tilde{I}_{\Gamma(\Lambda_1)} & \text{if } \gamma \in \Lambda_1 \setminus \Lambda_2 \\ \tilde{I}_{\Gamma(\Lambda_2)} & \text{if } \gamma \in \Lambda_2 \setminus \Lambda_1 \\ \bigcup_{\nu_1 \in \tilde{I}_{\Gamma(\Lambda_1)}, \nu_2 \in \tilde{I}_{\Gamma(\Lambda_2)}} \hat{\nu}_{12} \geq \max(\min(\tilde{I}_{\Gamma(\Lambda_1)}), \min(\tilde{I}_{\Gamma(\Lambda_2)})) & \text{if } \gamma \in \Lambda_2 \cap \Lambda_1 \end{cases} \quad (12)$$

$$\tilde{F}_{\Gamma(\Lambda_1) \cap \Gamma(\Lambda_2)} = \begin{cases} \tilde{F}_{\Gamma(\Lambda_1)} & \text{if } \gamma \in \Lambda_1 \setminus \Lambda_2 \\ \tilde{F}_{\Gamma(\Lambda_2)} & \text{if } \gamma \in \Lambda_2 \setminus \Lambda_1 \\ \bigcup_{\eta_1 \in \tilde{F}_{\Gamma(\Lambda_1)}, \eta_2 \in \tilde{F}_{\Gamma(\Lambda_2)}} \hat{\eta}_{12} \geq \max(\min(\tilde{F}_{\Gamma(\Lambda_1)}), \min(\tilde{F}_{\Gamma(\Lambda_2)})) & \text{if } \gamma \in \Lambda_2 \cap \Lambda_1 \end{cases} \quad (13)$$

Example 3.5. Considering Example 3.4,

Nguyen Thu Huong, Nguyen Tho Thong, Dao The Son, Dao Cao Son and Florentin Smarandache,
A Novel Multi-Criteria Decision-Making Approach for the Solution of Cigarette Use Reduction Based
on Generalized Neutrosophic Hypersoft Sets

$$(\Gamma, \Lambda_1) \cap (\Gamma, \Lambda_2) = \left\{ \begin{aligned} &\left\langle u_1, \left(\hat{\gamma}_1, \langle \{0.4, 0.5\}, \{0.3, 0.5, 0.6\}, \{0.5, 0.6\} \rangle, \hat{\gamma}_2, \langle \{0.7, 0.8\}, \{0.5, 0.6\}, \{0.4, 0.7\} \rangle \right) \right\rangle \\ &\left\langle u_2, \left(\hat{\gamma}_1, \langle \{0.4\}, \{0.2, 0.3\}, \{0.6\} \rangle, \hat{\gamma}_2, \langle \{0.4, 0.6\}, \{0.3\}, \{0.2\} \rangle \right) \right\rangle \\ &\left\langle u_3, \left(\hat{\gamma}_1, \langle \{0.6, 0.8\}, \{0.8, 0.9\}, \{0.3, 0.7\} \rangle, \hat{\gamma}_2, \langle \{0.8\}, \{0.1, 0.4\}, \{0.2\} \rangle \right) \right\rangle \end{aligned} \right\}$$

Definition 3.4. Let $\Gamma_1(\Lambda)$ and $\Gamma_2(\Lambda)$ be two GNHSN; $\lambda > 0$, and then the algebraic operational laws are defined as:

(1)

$$\Gamma_1(\Lambda) \oplus \Gamma_2(\Lambda) = \left\{ \begin{aligned} &\bigcup_{\forall \sigma_1 \in \tilde{T}(\Gamma_1(\Lambda)); \forall \sigma_2 \in \tilde{T}(\Gamma_2(\Lambda))} \sigma_1 + \sigma_2 - \sigma_1 \sigma_2, \\ &\bigcup_{\forall \nu_1 \in \tilde{I}(\Gamma_1(\Lambda)); \forall \nu_2 \in \tilde{I}(\Gamma_2(\Lambda))} \nu_1 \nu_2, \\ &\bigcup_{\forall \eta_1 \in \tilde{F}(\Gamma_1(\Lambda)); \forall \eta_2 \in \tilde{F}(\Gamma_2(\Lambda))} \eta_1 \eta_2 \end{aligned} \right\}$$

(2)

$$\Gamma_1(\Lambda) \otimes \Gamma_2(\Lambda) = \left\{ \begin{aligned} &\bigcup_{\forall \sigma_1 \in \tilde{T}(\Gamma_1(\Lambda)); \forall \sigma_2 \in \tilde{T}(\Gamma_2(\Lambda))} \sigma_1 \sigma_2, \\ &\bigcup_{\forall \nu_1 \in \tilde{I}(\Gamma_1(\Lambda)); \forall \nu_2 \in \tilde{I}(\Gamma_2(\Lambda))} \nu_1 + \nu_2 - \nu_1 \nu_2, \\ &\bigcup_{\forall \eta_1 \in \tilde{F}(\Gamma_1(\Lambda)); \forall \eta_2 \in \tilde{F}(\Gamma_2(\Lambda))} \eta_1 + \eta_2 - \eta_1 \eta_2 \end{aligned} \right\}$$

(3)

$$\lambda \Gamma_1(\Lambda) = \left\{ \begin{aligned} &\bigcup_{\forall \sigma_1 \in \tilde{T}(\Gamma_1(\Lambda))} 1 - (1 - \sigma_1)^\lambda, \quad \bigcup_{\forall \nu_1 \in \tilde{I}(\Gamma_1(\Lambda))} (\nu_1)^\lambda, \\ &\bigcup_{\forall \eta_1 \in \tilde{F}(\Gamma_1(\Lambda))} (\eta_1)^\lambda \end{aligned} \right\}$$

(4)

$$(\Gamma_1(\Lambda))^\lambda = \left\{ \begin{aligned} &\bigcup_{\forall \sigma_1 \in \tilde{T}(\Gamma_1(\Lambda))} (\sigma_1)^\lambda, \quad \bigcup_{\forall \nu_1 \in \tilde{I}(\Gamma_1(\Lambda))} 1 - (1 - \nu_1)^\lambda, \\ &\bigcup_{\forall \eta_1 \in \tilde{F}(\Gamma_1(\Lambda))} 1 - (1 - \eta_1)^\lambda \end{aligned} \right\}$$

Definition 3.5. Let $\Gamma_{ij}(\Lambda) = \{\langle \tilde{T}_{ij}(\Gamma(\Lambda)), \tilde{I}_{ij}(\Gamma(\Lambda)), \tilde{F}_{ij}(\Gamma(\Lambda)) \rangle\}$ be the GNHSN. Then the score function and accuracy function can be defined as followings

(1) Score function

$$S(\Gamma_{ij}(\Lambda)) = \frac{1}{3mn} \sum_{j=1}^m \sum_{i=1}^n \left(2 + \frac{1}{|\tilde{T}(\Gamma_{ij}(\Lambda))|} \sum_{\sigma \in \tilde{T}(\Gamma_{ij}(\Lambda))} (\sigma) - \frac{1}{|\tilde{I}(\Gamma_{ij}(\Lambda))|} \sum_{\nu \in \tilde{I}(\Gamma_{ij}(\Lambda))} (\nu) - \frac{1}{|\tilde{F}(\Gamma_{ij}(\Lambda))|} \sum_{\eta \in \tilde{F}(\Gamma_{ij}(\Lambda))} (\eta) \right) \quad (14)$$

(2) Accuracy function

$$A(\Gamma_{ij}(\Lambda)) = \frac{1}{3mn} \sum_{j=1}^m \sum_{i=1}^n \left(\frac{1}{|\tilde{T}(\Gamma_{ij}(\Lambda))|} \sum_{\sigma \in \tilde{T}(\Gamma_{ij}(\Lambda))} (\sigma) + \frac{1}{|\tilde{I}(\Gamma_{ij}(\Lambda))|} \sum_{\nu \in \tilde{I}(\Gamma_{ij}(\Lambda))} (\nu) + \frac{1}{|\tilde{F}(\Gamma_{ij}(\Lambda))|} \sum_{\eta \in \tilde{F}(\Gamma_{ij}(\Lambda))} (\eta) \right) \quad (15)$$

Where $|\tilde{T}(\Gamma_{ij}(\Lambda))|$, $|\tilde{I}(\Gamma_{ij}(\Lambda))|$ and $|\tilde{F}(\Gamma_{ij}(\Lambda))|$ is power of the corresponding sets

Example 3.6. Considering Example 3.1,

Let

$$(\Gamma, \Lambda) = \left\{ \langle u_1, (\hat{\gamma}_1 \langle \{0.1, 0.5\}, \{0.3, 0.6\}, \{0.4, 0.5\}\rangle, \hat{\gamma}_2 \langle \{0.8\}, \{0.2, 0.6\}, \{0.1, 0.3, 0.7\}\rangle) \rangle, \right. \\ \left. \langle u_2, (\hat{\gamma}_3 \langle \{0.3, 0.7\}, \{0.1, 0.5\}, \{0.5, 0.8\}\rangle, \hat{\gamma}_4 \langle \{0.5, 0.8\}, \{0.3, 0.8\}, \{0.2, 0.6\}\rangle) \rangle \right\}$$

$$S(\Gamma, \Lambda) = 0.273; A(\Gamma, \Lambda) = 0.010$$

Definition 3.6. Let Γ_{ij}^A and Γ_{ij}^B be two GNHSN.

Hamming distance

$$d(\Gamma_{ij}^A, \Gamma_{ij}^B) = \frac{1}{3mn} \sum_{j=1}^m \sum_{i=1}^n \left(\frac{1}{l_{\tilde{T}}} \sum_{\tau=1}^{l_{\tilde{T}}} |\sigma_{ij}^A(\tau) - \sigma_{ij}^B(\tau)| + \frac{1}{l_{\tilde{I}}} \sum_{\tau=1}^{l_{\tilde{I}}} |\nu_{ij}^A(\tau) - \nu_{ij}^B(\tau)| + \frac{1}{l_{\tilde{F}}} \sum_{\tau=1}^{l_{\tilde{F}}} |\eta_{ij}^A(\tau) - \eta_{ij}^B(\tau)| \right) \quad (16)$$

Euclidean distance

$$d(\Gamma_{ij}^A, \Gamma_{ij}^B) = \frac{1}{3mn} \sum_{j=1}^m \sum_{i=1}^n \left(\sqrt{\left(\frac{1}{l_{\tilde{T}}} \sum_{\tau=1}^{l_{\tilde{T}}} (\sigma_{ij}^A(\tau) - \sigma_{ij}^B(\tau))^2 + \frac{1}{l_{\tilde{I}}} \sum_{\tau=1}^{l_{\tilde{I}}} (\nu_{ij}^A(\tau) - \nu_{ij}^B(\tau))^2 + \frac{1}{l_{\tilde{F}}} \sum_{\tau=1}^{l_{\tilde{F}}} (\eta_{ij}^A(\tau) - \eta_{ij}^B(\tau))^2 \right)} \right) \quad (17)$$

Where $\sigma_{ij}^A \in \tilde{T}_{ij}^A$; $\sigma_{ij}^B \in \tilde{T}_{ij}^B$; $\nu_{ij}^A \in \tilde{I}_{ij}^B$; $\nu_{ij}^B \in \tilde{I}_{ij}^B$; $\eta_{ij}^A \in \tilde{F}_{ij}^A$; $\eta_{ij}^B \in \tilde{F}_{ij}^B$ are the τ th largest values of truth-membership hesitant degrees, indeterminacy-membership hesitant degrees, and falsity membership hesitant degrees of A and B , respectively.

Definition 3.7. Let $\tilde{\Gamma}$ be a set of GNHSN; $\Gamma_{ij} \in \tilde{\Gamma}$; $\Gamma_{ij}(\Lambda) = \{\langle \tilde{T}_{ij}(\Gamma(\Lambda)); i = 1, 2, 3, \dots, n; j = 1, 2, 3, \dots, m; \omega_i \text{ and } w_j \text{ be the weights for experts and multi sub-attributes of the deliberated attributes consistently along with indicated circumstances } \sum_{j=1}^n \omega_i = 1 \text{ and } \sum_{j=1}^n w_i = 1. \text{ Then Generalized Neutrosophic Hypersoft Weighted Aggregation Operator (GNHWAO) can be defined as$

$$GNHWAO(\tilde{\Gamma}) = \bigoplus_{j=1}^m w_j \bigoplus_{i=1}^n \omega_i \Gamma_{ij}(\Lambda) \quad (18)$$

Theorem 3.7. Let $\tilde{\Gamma}$ be a set of GNHSN, then, obtained aggregated values using equation (19) is an GNHSN and

$$GNHWAO(\tilde{\Gamma}) = \left\langle \bigcup_{\sigma_{ij} \in \tilde{T}(\Gamma_{ij}(\Lambda))} \left\{ 1 - \prod_{j=1}^m \left(\prod_{i=1}^n (1 - \sigma_{ij})^{\omega_i} \right)^{w_j} \right\}, \right. \\ \left. \bigcup_{\nu_{ij} \in \tilde{I}(\Gamma_{ij}(\Lambda))} \left\{ \prod_{j=1}^m \left(\prod_{i=1}^n (\nu_{ij})^{\omega_i} \right)^{w_j} \right\}, \right. \\ \left. \bigcup_{\eta_{ij} \in \tilde{F}(\Gamma_{ij}(\Lambda))} \left\{ \prod_{j=1}^m \left(\prod_{i=1}^n (\eta_{ij})^{\omega_i} \right)^{w_j} \right\} \right\rangle \quad (19)$$

Proof: Using the principle of mathematical induction, GNHWAO operator can be proved by utilizing the following steps,

For $n = 1$, we get $\omega_1 = 1$, then we have

$$GNHWAO(\tilde{\Gamma}) = GNHWAO(\Gamma_{11}(\Lambda), \Gamma_{12}(\Lambda), \dots, \Gamma_{1m}(\Lambda)) = \bigoplus_{j=1}^m w_j \Gamma_{1j}(\Lambda) \\ = \left\langle \bigcup_{\sigma_{1j} \in \tilde{T}(\Gamma_{1j}(\Lambda))} \left\{ 1 - \prod_{j=1}^m (1 - \sigma_{1j})^{w_j} \right\}, \bigcup_{\nu_{1j} \in \tilde{I}(\Gamma_{1j}(\Lambda))} \left\{ \prod_{j=1}^m (\nu_{1j})^{w_j} \right\}, \right. \\ \left. \bigcup_{\eta_{1j} \in \tilde{F}(\Gamma_{1j}(\Lambda))} \left\{ \prod_{j=1}^m (\eta_{1j})^{w_j} \right\} \right\rangle \\ = \left\langle \bigcup_{\sigma_{1j} \in \tilde{T}(\Gamma_{1j}(\Lambda))} \left\{ 1 - \prod_{j=1}^m \left(\prod_{i=1}^1 (1 - \sigma_{ij})^1 \right)^{w_j} \right\}, \right. \\ \left. \bigcup_{\nu_{1j} \in \tilde{I}(\Gamma_{1j}(\Lambda))} \left\{ \prod_{j=1}^m \left(\prod_{i=1}^1 (\nu_{ij})^1 \right)^{w_j} \right\}, \bigcup_{\eta_{1j} \in \tilde{F}(\Gamma_{1j}(\Lambda))} \left\{ \prod_{j=1}^m \left(\prod_{i=1}^1 (\eta_{ij})^1 \right)^{w_j} \right\} \right\rangle$$

For $m = 1$, we get $w_1 = 1$, Then, we have,

$$\begin{aligned}
 GNHWA O(\tilde{\Gamma}) &= GNHWA O(\Gamma_{11}(\Lambda), \Gamma_{21}(\Lambda), \dots, \Gamma_{n1}(\Lambda)) = \bigoplus_{i=1}^n \omega_i \Gamma_{i1}(\Lambda) \\
 &= \left\langle \bigcup_{\sigma_{i1} \in \tilde{T}(\Gamma_{i1}(\Lambda))} \left\{ 1 - \prod_{i=1}^n (1 - \sigma_{i1})^{\omega_i} \right\}, \bigcup_{\nu_{i1} \in \tilde{I}(\Gamma_{i1}(\Lambda))} \left\{ \prod_{i=1}^n (\nu_{i1})^{\omega_i} \right\}, \right. \\
 &\quad \left. \bigcup_{\eta_{i1} \in \tilde{F}(\Gamma_{i1}(\Lambda))} \left\{ \prod_{i=1}^n (\eta_{i1})^{\omega_i} \right\} \right\rangle \\
 &= \left\langle \bigcup_{\sigma_{i1} \in \tilde{T}(\Gamma_{i1}(\Lambda))} \left\{ 1 - \prod_{j=1}^1 \left(\prod_{i=1}^n (1 - \sigma_{ij})^{\omega_i} \right)^1 \right\}, \bigcup_{\nu_{i1} \in \tilde{I}(\Gamma_{i1}(\Lambda))} \left\{ \prod_{j=1}^1 \left(\prod_{i=1}^n (\nu_{ij})^{\omega_i} \right)^1 \right\}, \right. \\
 &\quad \left. \bigcup_{\eta_{i1} \in \tilde{F}(\Gamma_{i1}(\Lambda))} \left\{ \prod_{j=1}^1 \left(\prod_{i=1}^n (\eta_{ij})^{\omega_i} \right)^1 \right\} \right\rangle
 \end{aligned}$$

So, $n = 1$ and $m = 1$, equation (19) satisfied. Consider equation (19) holds for $n = \phi + 1$; $m = \varphi$, such as,

$$\begin{aligned}
 \bigoplus_{j=1}^{\varphi+1} w_j \bigoplus_{i=1}^{\phi+1} \omega_i \Gamma_{ij}(\Lambda) &= \left(\bigoplus_{j=1}^{\varphi} w_j \bigoplus_{i=1}^{\phi+1} \omega_i \Gamma_{ij}(\Lambda) \right) \oplus \left(\bigoplus_{j=\varphi+1}^{\varphi+1} w_j \bigoplus_{i=1}^{\phi+1} \omega_i \Gamma_{ij}(\Lambda) \right) \\
 &= \left\langle \bigcup_{\sigma_{ij} \in \tilde{T}(\Gamma_{ij}(\Lambda))} \left\{ 1 - \prod_{j=1}^{\varphi} \left(\prod_{i=1}^{\phi+1} (1 - \sigma_{ij})^{\omega_i} \right)^{w_j} \right\}, \right. \\
 &\quad \left. \bigcup_{\nu_{ij} \in \tilde{I}(\Gamma_{ij}(\Lambda))} \left\{ \prod_{j=1}^{\varphi} \left(\prod_{i=1}^{\phi+1} (\nu_{ij})^{\omega_i} \right)^{w_j} \right\}, \bigcup_{\eta_{ij} \in \tilde{F}(\Gamma_{ij}(\Lambda))} \left\{ \prod_{j=1}^{\varphi} \left(\prod_{i=1}^{\phi+1} (\eta_{ij})^{\omega_i} \right)^{w_j} \right\} \right\rangle \\
 &\quad \oplus \left\langle \bigcup_{\sigma_{ij} \in \tilde{T}(\Gamma_{ij}(\Lambda))} \left\{ 1 - \prod_{j=\varphi+1}^{\varphi+1} \left(\prod_{i=1}^{\phi+1} (1 - \sigma_{ij})^{\omega_i} \right)^{w_j} \right\}, \right. \\
 &\quad \left. \bigcup_{\nu_{ij} \in \tilde{I}(\Gamma_{ij}(\Lambda))} \left\{ \prod_{j=\varphi+1}^{\varphi+1} \left(\prod_{i=1}^{\phi+1} (\nu_{ij})^{\omega_i} \right)^{w_j} \right\}, \bigcup_{\eta_{ij} \in \tilde{F}(\Gamma_{ij}(\Lambda))} \left\{ \prod_{j=\varphi+1}^{\varphi+1} \left(\prod_{i=1}^{\phi+1} (\eta_{ij})^{\omega_i} \right)^{w_j} \right\} \right\rangle \\
 &= \left\langle \bigcup_{\sigma_{ij} \in \tilde{T}(\Gamma_{ij}(\Lambda))} \left\{ 1 - \prod_{j=1}^{\varphi+1} \left(\prod_{i=1}^{\phi+1} (1 - \sigma_{ij})^{\omega_i} \right)^{w_j} \right\}, \right. \\
 &\quad \left. \bigcup_{\nu_{ij} \in \tilde{I}(\Gamma_{ij}(\Lambda))} \left\{ \prod_{j=1}^{\varphi+1} \left(\prod_{i=1}^{\phi+1} (\nu_{ij})^{\omega_i} \right)^{w_j} \right\}, \bigcup_{\eta_{ij} \in \tilde{F}(\Gamma_{ij}(\Lambda))} \left\{ \prod_{j=1}^{\varphi+1} \left(\prod_{i=1}^{\phi+1} (\eta_{ij})^{\omega_i} \right)^{w_j} \right\} \right\rangle
 \end{aligned}$$

and $n = \phi$; $m = \varphi + 1$, prove the same as above.

Therefore, it is correct for $n = \phi + 1$; $m = \varphi + 1$, Theorem 3.7 is proved.

Definition 3.8. Let $\tilde{\Gamma}$ be a set of GNHSN; $\Gamma_{ij} \in \tilde{\Gamma}$; $\Gamma_{ij}(\Lambda) = \{\langle \tilde{T}_{ij}(\Gamma(\Lambda)); i = 1, 2, 3, \dots, n; j = 1, 2, 3, \dots, m; \omega_i \text{ and } w_j \text{ be the weights for experts and multi sub-attributes of the deliberated attributes consistently along with indicated circumstances } \sum_{j=1}^n \omega_i = 1 \text{ and } \sum_{j=1}^n w_i = 1. \text{ Then Generalized Neutrosophic Hypersoft Weighted Aggregation Geometry (GNHWAG) can be defined as$

$$GNHWAG(\tilde{\Gamma}) = \bigotimes_{j=1}^m w_j \bigotimes_{i=1}^n \omega_i \Gamma_{ij}(\Lambda) \quad (20)$$

Theorem 3.8. Let $\tilde{\Gamma}$ be a set of GNHSN, then, obtained aggregated values using equation (21) is an GNHSN and

$$GNHWAG(\tilde{\Gamma}) = \left\langle \bigcup_{\sigma_{ij} \in \tilde{T}(\Gamma_{ij}(\Lambda))} \left\{ \prod_{j=1}^m \left(\prod_{i=1}^n (\sigma_{ij})^{\omega_i} \right)^{w_j} \right\}, \right. \quad (21)$$

$$\left. \bigcup_{\nu_{ij} \in \tilde{I}(\Gamma_{ij}(\Lambda))} \left\{ 1 - \prod_{j=1}^m \left(\prod_{i=1}^n (1 - \nu_{ij})^{\omega_i} \right)^{w_j} \right\}, \right.$$

$$\left. \bigcup_{\eta_{ij} \in \tilde{F}(\Gamma_{ij}(\Lambda))} \left\{ 1 - \prod_{j=1}^m \left(\prod_{i=1}^n (1 - \eta_{ij})^{\omega_i} \right)^{w_j} \right\} \right\rangle$$

Proof: Proof is similar to Theorem 3.7.

4. A novel MCDM based on GNHSS

Input: Consider a multi-attribute decision-making problem based on generalized neutrosophic hypersoft sets (GNHSSs) where $\tilde{P} = \{P_1, \dots, P_n\}$; $P_r \in \tilde{P}$; $r = 1, 2, \dots, n$ represents set of alternatives and $\tilde{A} = \{A_1, \dots, A_m\}$ represents the sets of attributes with the corresponding attributes values are respectively the set $\tilde{L} = \{L_1, \dots, L_m\}$; Suppose that $\tilde{U} = \{K_1, K_2, \dots, K_s\}$; $d = 1, \dots, s$ represents the set of s decision makers. The attributes can be specified as a set $\Lambda = L_1 \times L_2 \times \dots \times L_m$. Let m-tuple $\hat{\gamma}_j \in \Lambda$; $j = 1, 2, \dots, z$; Let w_j be the weight of sub attributes relationships and ω_i be the weight of decision maker.

Output: Ranking alternatives

GNHSS – TOPSIS method

Step 1: Construct an GNHSM $\left[X_{\Gamma_{ij}(\Lambda)}^r \right]_{z \times s}$ by GNHSN. Let each m-tupe $\hat{\gamma}_j = \{l_1, l_2, \dots, l_m\} \in \Lambda$; $j = 1, 2, \dots, z$; $\left[X_{\Gamma_{ij}(\Lambda)}^r \right]_{z \times s} = \langle \tilde{T}_i^r(\hat{\gamma}_j), \tilde{I}_i^r(\hat{\gamma}_j), \tilde{F}_i^r(\hat{\gamma}_j) \rangle$, where $\tilde{T}_i^r(\hat{\gamma}_j), \tilde{I}_i^r(\hat{\gamma}_j), \tilde{F}_i^r(\hat{\gamma}_j)$ are respectively estimates of the decision-maker i for alternative r through relationship j , with some values in $[0, 1]$,

Step 2: Using equation (19), calculate the average ratings for each alternatives

$$\bar{P}^r = \bigoplus_{j=1}^m w_j \bigoplus_{i=1}^n \omega_i \langle \tilde{T}_i^r(\hat{\gamma}_j), \tilde{I}_i^r(\hat{\gamma}_j), \tilde{F}_i^r(\hat{\gamma}_j) \rangle = \langle T_{\bar{P}}^r, I_{\bar{P}}^r, F_{\bar{P}}^r \rangle \quad (22)$$

Where

$$\begin{aligned} T_{\bar{P}}^r &= \bigcup_{\sigma_{ij} \in \tilde{T}_i^r(\hat{\gamma}_j)} \left\{ 1 - \prod_{j=1}^m \left(\prod_{i=1}^n (1 - \sigma_{ij})^{\omega_i} \right)^{w_j} \right\}, \\ I_{\bar{P}}^r &= \bigcup_{\nu_{ij} \in \tilde{I}_i^r(\hat{\gamma}_j)} \left\{ \prod_{j=1}^m \left(\prod_{i=1}^n (\nu_{ij})^{\omega_i} \right)^{w_j} \right\}, \\ F_{\bar{P}}^r &= \bigcup_{\eta_{ij} \in \tilde{F}_i^r(\hat{\gamma}_j)} \left\{ \prod_{j=1}^m \left(\prod_{i=1}^n (\eta_{ij})^{\omega_i} \right)^{w_j} \right\} \end{aligned}$$

Step 3: Determination of PIS P^+ , NIS P^- and calculate d^+ , d^-

$$P^+ = \langle \max((T_{\bar{P}}^r)^+), \min((I_{\bar{P}}^r)^-), \min((F_{\bar{P}}^r)^-) \rangle; \quad (23)$$

$$P^- = \langle \min((T_{\bar{P}}^r)^+), \max((I_{\bar{P}}^r)^-), \max((F_{\bar{P}}^r)^-) \rangle;$$

$$(d^r)^+ = \sqrt{(\bar{P}^r - P^+)^2}; (d^r)^- = \sqrt{(\bar{P}^r - P^-)^2} \quad (24)$$

Step 4: Determine the relative closeness coefficient to best condition

$$RCC^r = \frac{(d^r)^+}{(d^r)^+ + (d^r)^-} \quad (25)$$

Step 5: Rank the options according to their respective closeness coefficients. The most acceptable option is determined based on the relative closeness coefficients, with the option having the highest closeness coefficient value being selected.

5. APPLICATION

5.1. Decision-making model for cigarettes use reduction

Investing in tobacco control is a strategic move that offers substantial public health, economic, and social benefits. It is a cost-effective approach that not only saves lives but also enhances economic productivity and supports global health objectives. By prioritizing tobacco control, governments and organizations can achieve significant returns on investment while contributing to a healthier, more sustainable future.

This model presents an innovative Multiple Criteria Decision-Making (MCDM) approach designed to reduce cigarette usage. It addresses not only the complexities of decision-making in cigarette use reduction strategies, but also provides a nuanced analysis of various factors influencing smoking behavior. This approach emphasizes the integration of diverse criteria and uncertainties in evaluating and implementing smoking reduction policies, making itself as a robust tool for policymakers and health organizations aiming to mitigate smoking-related

issues. Policies packages to reduce tobacco prevalence, such as MPOWER, introduced in 2008 by the WHO FCTC, have shown steady progress in implementation all over the world. Among these, tax policy (P_1) is considered the most cost-effective measure, smoke-free air (P_2) saves lives and benefits businesses and economies, cessation treatment (P_3) can more than double the chances of quitting successfully, and has more impact when combined with other tobacco control strategies (such as health warnings – P_4 and marketing restriction – P_5 - including banning advertising, promotion, and sponsorship). Mass media anti-tobacco campaigns (P_6), if well-designed and impactful, can reduce tobacco use, boost quit attempts, lower youth initiation rates and minimize second-hand smoke exposure. Regarding implementation, to restrict younger or underage consumers (youth access restrictions – P_7) particularly those buy around educational institutes, policymakers need to enforce retail management P_8 such as licensing & density to reduce the availability and accessibility of tobacco products. These measures are evaluated using 15 criteria divided into 5 groups (Attractiveness, Affordability, Accessibility, Awareness, Environment)

(1) Attractiveness - C_1 : Tobacco companies have taken advantage of cigarettes' addictive potential through changes in content and design to attract and retain smokers, to get children and adolescents exposed to their brands (brand exposure - C_{11}), to reduce tobacco user's perception of risk and harm (brand perception - C_{12}), and to redesign their products (product designs - C_{13}) to sustain profitability.

(2) Affordability - C_2 : Increasing the current price of cigarettes (price, current - C_{21}) through taxation directly affects their affordability. Higher prices (price, change - C_{22}) discourage smoking initiation and promote cessation. Frequent price hikes make smoking progressively less affordable over time, deterring long-term use. Greater distances to purchase points (Distance to the point of sale - C_{23}) reduce convenience, leading to lower smoking rates among youth and casual smokers.

(3) Accessibility - C_3 : Effective enforcement of sales restrictions at points of sale can significantly reduce smoking rates by limiting the availability of tobacco to underage individuals. Easy access to tobacco products (Easiness to locate the point of sale - C_{31}) increases consumption, especially among youth; limiting retailers and enforcing zoning laws can curb smoking rates. Proximity to retailers (Distance to the point of sale - C_{32}) influences purchase frequency; policies requiring minimum distances from schools can reduce use. Restrictions at points of sale (Restrictions at point of sale - C_{33}), such as age verification, advertising bans, and product display bans, deter impulse purchases and prevent youth access.

(4) Awareness - C_4 : Awareness of health risks (Understand about harms - C_{41}) and societal costs drive the urgency for stringent measures like taxation and advertising bans. Acknowledging the ethical obligation (Understand about responsibility - C_{42}) to protect public health

TABLE 1. Determine Relationships of sub-attributes

Symbol	Sub-criteria's Relationships	Symbol	Sub-criteria's Relationships
R_1	$C_{11} \times C_{22} \times C_{32} \times C_{41} \times C_{52}$	R_6	$C_{12} \times C_{21} \times C_{32} \times C_{41} \times C_{52}$
R_2	$C_{12} \times C_{21} \times C_{31} \times C_{42} \times C_{53}$	R_7	$C_{12} \times C_{22} \times C_{33} \times C_{41} \times C_{51}$
R_3	$C_{11} \times C_{22} \times C_{33} \times C_{42} \times C_{51}$	R_8	$C_{11} \times C_{22} \times C_{33} \times C_{42} \times C_{51}$
R_4	$C_{13} \times C_{21} \times C_{32} \times C_{42} \times C_{52}$	R_9	$C_{12} \times C_{22} \times C_{31} \times C_{42} \times C_{52}$
R_5	$C_{11} \times C_{22} \times C_{31} \times C_{43} \times C_{53}$	R_{10}	$C_{13} \times C_{22} \times C_{32} \times C_{41} \times C_{53}$

ensures that policymakers prioritize tobacco control, fostering accountability among stakeholders. Knowledge of viable alternatives (Understand about alternatives – C_{43}), such as cessation programs, enables the creation of supportive policies that help smokers quit. This approach ensures policies are supportive rather than merely punitive.

(5) Environment – C_5 : Social stigma, peer pressure, and family influence significantly impact tobacco control policy decisions. Social stigma (Social stigma – C_{51}) discourages smoking through negative perceptions, supporting stricter regulations like public smoking bans. Peer pressure (Peer pressure – C_{52}) particularly among youth, necessitates educational programs and targeted campaigns to prevent smoking initiation. Family influence (Family influence – C_{53}) shapes smoking habits, making family-based interventions and parental education crucial for effective prevention and cessation.

5.2. Experiment for Solution approach using the GNHSS-TOPSIS

Figure 1 shows the implementation steps for the solution of cigarette use reduction using the GNHSS-TOPSIS.

Step 1: Build a decision-making matrix for the solution of cigarette use reduction. Two experts, K_1 and K_2 , specializing in the tobacco control domain, were selected to identify the connections between the sub-criteria and estimated values essential for decision-making. Table 1 shows the relationships among the sub-attributes in the issue of reducing cigarette use.

Tables 2, 3 shows the estimated values of K_1, K_2 for policies through relationships

Steps 2 & 3: Calculate the average value for the policies based on the experts' assessments using equation (22) and apply equation (23) to determine PIS and NIS. The values of PIS and NIS are given below:

$$PIS = \langle 0.64163, 0.48698, 0.0.48698 \rangle; NIS = \langle 0.48698, 0.64163, 0.64163 \rangle;$$

Next, the distance values of 8 policies to PIS and NIS are calculated using equation (24). Table 4 shows the distance values of policies to PIS and NIS.

Step 4: Use equation (25) to calculate the correlation coefficient of policies. Table 5 shows the correlation coefficient values of 8 policies.

TABLE 2. Evaluate of K_1 by hesitant neutrosophic number

	P_1	P_2	P_3	P_4
R_1	$\langle \{0.4, 0.65\}, \{0.5, 0.25\}, \{0.55, 0.25\} \rangle$	$\langle \{0.55, 0.65\}, \{0.45, 0.25\}, \{0.35, 0.25\} \rangle$	$\langle \{0.65\}, \{0.25\}, \{0.25\} \rangle$	$\langle \{0.55, 0.65\}, \{0.45, 0.25\}, \{0.35, 0.25\} \rangle$
R_2	$\langle \{0.4, 0.65\}, \{0.5, 0.25\}, \{0.55, 0.25\} \rangle$	$\langle \{0.55, 0.65\}, \{0.45, 0.25\}, \{0.35, 0.25\} \rangle$	$\langle \{0.55, 0.65\}, \{0.45, 0.25\}, \{0.35, 0.25\} \rangle$	$\langle \{0.55\}, \{0.45\}, \{0.35\} \rangle$
R_3	$\langle \{0.4, 0.65\}, \{0.5, 0.25\}, \{0.55, 0.25\} \rangle$	$\langle \{0.55, 0.65\}, \{0.45, 0.25\}, \{0.35, 0.25\} \rangle$	$\langle \{0.55, 0.65\}, \{0.45, 0.25\}, \{0.35, 0.25\} \rangle$	$\langle \{0.55, 0.65\}, \{0.45, 0.25\}, \{0.35, 0.25\} \rangle$
R_4	$\langle \{0.65\}, \{0.25\}, \{0.25\} \rangle$	$\langle \{0.55, 0.65\}, \{0.45, 0.25\}, \{0.35, 0.25\} \rangle$	$\langle \{0.25, 0.55\}, \{0.55, 0.45\}, \{0.65, 0.35\} \rangle$	$\langle \{0.4, 0.55\}, \{0.5, 0.45\}, \{0.55, 0.35\} \rangle$
R_5	$\langle \{0.4, 0.65\}, \{0.5, 0.25\}, \{0.55, 0.25\} \rangle$	$\langle \{0.4, 0.65\}, \{0.5, 0.25\}, \{0.55, 0.25\} \rangle$	$\langle \{0.55, 0.65\}, \{0.45, 0.25\}, \{0.35, 0.25\} \rangle$	$\langle \{0.4, 0.65\}, \{0.5, 0.25\}, \{0.55, 0.25\} \rangle$
R_6	$\langle \{0.4, 0.65\}, \{0.5, 0.25\}, \{0.55, 0.25\} \rangle$	$\langle \{0.55, 0.65\}, \{0.45, 0.25\}, \{0.35, 0.25\} \rangle$	$\langle \{0.65\}, \{0.25\}, \{0.25\} \rangle$	$\langle \{0.55\}, \{0.45\}, \{0.35\} \rangle$
R_7	$\langle \{0.4, 0.65\}, \{0.5, 0.25\}, \{0.55, 0.25\} \rangle$	$\langle \{0.55, 0.65\}, \{0.45, 0.25\}, \{0.35, 0.25\} \rangle$	$\langle \{0.55, 0.65\}, \{0.45, 0.25\}, \{0.35, 0.25\} \rangle$	$\langle \{0.55\}, \{0.45\}, \{0.35\} \rangle$
R_8	$\langle \{0.4, 0.65\}, \{0.5, 0.25\}, \{0.55, 0.25\} \rangle$	$\langle \{0.55, 0.65\}, \{0.45, 0.25\}, \{0.35, 0.25\} \rangle$	$\langle \{0.55, 0.65\}, \{0.45, 0.25\}, \{0.35, 0.25\} \rangle$	$\langle \{0.55, 0.65\}, \{0.45, 0.25\}, \{0.35, 0.25\} \rangle$
R_9	$\langle \{0.4, 0.65\}, \{0.5, 0.25\}, \{0.55, 0.25\} \rangle$	$\langle \{0.55, 0.65\}, \{0.45, 0.25\}, \{0.35, 0.25\} \rangle$	$\langle \{0.55, 0.65\}, \{0.45, 0.25\}, \{0.35, 0.25\} \rangle$	$\langle \{0.55\}, \{0.45\}, \{0.35\} \rangle$
R_{10}	$\langle \{0.4, 0.65\}, \{0.5, 0.25\}, \{0.55, 0.25\} \rangle$	$\langle \{0.55, 0.65\}, \{0.45, 0.25\}, \{0.35, 0.25\} \rangle$	$\langle \{0.65\}, \{0.25\}, \{0.25\} \rangle$	$\langle \{0.4, 0.55\}, \{0.5, 0.45\}, \{0.55, 0.35\} \rangle$
	P_5	P_6	P_7	P_8
R_1	$\langle \{0.4, 0.65\}, \{0.5, 0.25\}, \{0.55, 0.25\} \rangle$	$\langle \{0.4, 0.65\}, \{0.5, 0.25\}, \{0.55, 0.25\} \rangle$	$\langle \{0.4, 0.55\}, \{0.5, 0.45\}, \{0.55, 0.35\} \rangle$	$\langle \{0.25, 0.4\}, \{0.55, 0.5\}, \{0.65, 0.55\} \rangle$
R_2	$\langle \{0.4, 0.55\}, \{0.5, 0.45\}, \{0.55, 0.35\} \rangle$	$\langle \{0.55\}, \{0.45\}, \{0.35\} \rangle$	$\langle \{0.4, 0.55\}, \{0.5, 0.45\}, \{0.55, 0.35\} \rangle$	$\langle \{0.25, 0.4\}, \{0.55, 0.5\}, \{0.65, 0.55\} \rangle$
R_3	$\langle \{0.4, 0.55, 0.65\}, \{0.5, 0.45, 0.25\}, \{0.55, 0.35, 0.25\} \rangle$	$\langle \{0.4, 0.55\}, \{0.5, 0.45\}, \{0.55, 0.35\} \rangle$	$\langle \{0.55\}, \{0.45\}, \{0.35\} \rangle$	$\langle \{0.25, 0.4\}, \{0.55, 0.5\}, \{0.65, 0.55\} \rangle$
R_4	$\langle \{0.4, 0.55\}, \{0.5, 0.45\}, \{0.55, 0.35\} \rangle$	$\langle \{0.4, 0.55\}, \{0.5, 0.45\}, \{0.55, 0.35\} \rangle$	$\langle \{0.4, 0.55\}, \{0.5, 0.45\}, \{0.55, 0.35\} \rangle$	$\langle \{0.25, 0.4\}, \{0.55, 0.5\}, \{0.65, 0.55\} \rangle$
R_5	$\langle \{0.4\}, \{0.5\}, \{0.55\} \rangle$	$\langle \{0.55, 0.65\}, \{0.45, 0.25\}, \{0.35, 0.25\} \rangle$	$\langle \{0.4, 0.55\}, \{0.5, 0.45\}, \{0.55, 0.35\} \rangle$	$\langle \{0.25\}, \{0.55\}, \{0.65\} \rangle$
R_6	$\langle \{0.4, 0.65\}, \{0.5, 0.25\}, \{0.55, 0.25\} \rangle$	$\langle \{0.4, 0.65\}, \{0.5, 0.25\}, \{0.55, 0.25\} \rangle$	$\langle \{0.4, 0.55\}, \{0.5, 0.45\}, \{0.55, 0.35\} \rangle$	$\langle \{0.4\}, \{0.5\}, \{0.55\} \rangle$
R_7	$\langle \{0.65\}, \{0.25\}, \{0.25\} \rangle$	$\langle \{0.4, 0.65\}, \{0.5, 0.25\}, \{0.55, 0.25\} \rangle$	$\langle \{0.55\}, \{0.45\}, \{0.35\} \rangle$	$\langle \{0.4\}, \{0.5\}, \{0.55\} \rangle$
R_8	$\langle \{0.4, 0.55, 0.65\}, \{0.5, 0.45, 0.25\}, \{0.55, 0.35, 0.25\} \rangle$	$\langle \{0.4, 0.55\}, \{0.5, 0.45\}, \{0.55, 0.35\} \rangle$	$\langle \{0.55\}, \{0.45\}, \{0.35\} \rangle$	$\langle \{0.25, 0.4\}, \{0.55, 0.5\}, \{0.65, 0.55\} \rangle$
R_9	$\langle \{0.4, 0.55\}, \{0.5, 0.45\}, \{0.55, 0.35\} \rangle$	$\langle \{0.4, 0.55\}, \{0.5, 0.45\}, \{0.55, 0.35\} \rangle$	$\langle \{0.4, 0.55\}, \{0.5, 0.45\}, \{0.55, 0.35\} \rangle$	$\langle \{0.25\}, \{0.55\}, \{0.65\} \rangle$
R_{10}	$\langle \{0.55, 0.65\}, \{0.45, 0.25\}, \{0.35, 0.25\} \rangle$	$\langle \{0.55, 0.65\}, \{0.45, 0.25\}, \{0.35, 0.25\} \rangle$	$\langle \{0.55\}, \{0.45\}, \{0.35\} \rangle$	$\langle \{0.25, 0.4\}, \{0.55, 0.5\}, \{0.65, 0.55\} \rangle$

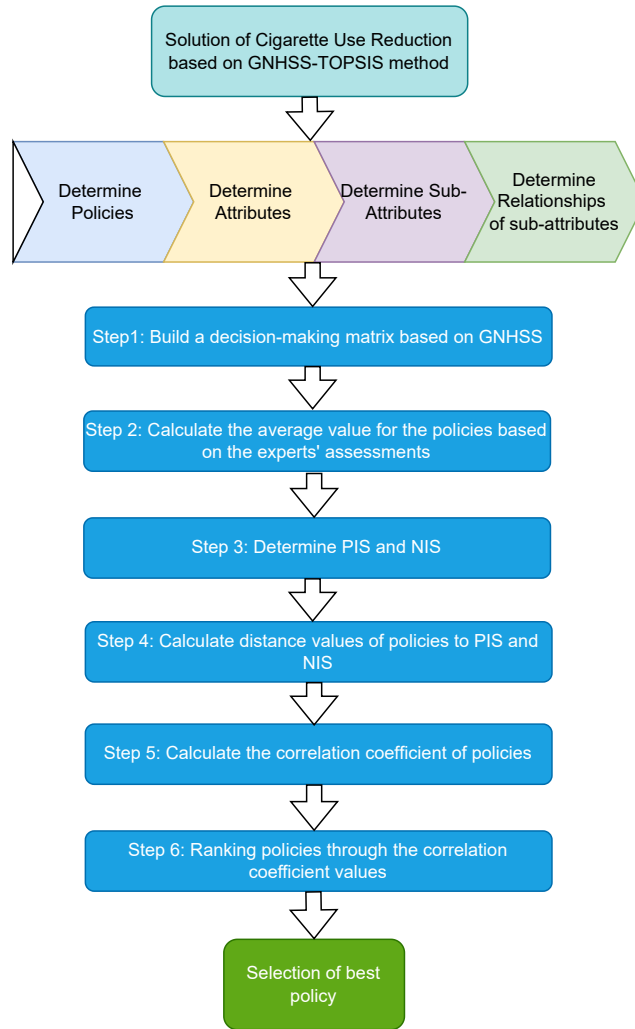


FIGURE 1. Solution of Cigarette Use Reduction based on GNHSS-TOPSIS

Step 5: Ranking policies through the correlation coefficient values in Table 5 and Figure 2. The ranking of the 8 policies is as follows $P_8 \succ P_5 \succ P_1 \succ P_6 \succ P_7 \succ P_4 \succ P_3 \succ P_2$ and P_8 is identified as the best policy for addressing the cigarette use reduction problem.

5.3. Comparison and Discussion

In this section, we demonstrate the effectiveness of the proposed model by contrasting its performance with the multi-criteria decision-making approaches of Saqlain et al. [29] and Zulqarnain et al. [36] in neutrosophic hypersoftset environments. The dataset estimated in Section 5.2 for reducing cigarette use is transformed into the structural format of the

TABLE 3. Evaluate of K_2 by hesitant neutrosophic number

	$\langle P_1 \rangle$	$\langle P_2 \rangle$	$\langle P_3 \rangle$	$\langle P_4 \rangle$
R_1	$\langle \{0.65\}, \{0.25\}, \{0.25\} \rangle$	$\langle \{0.55\}, \{0.45\}, \{0.35\} \rangle$	$\langle \{0.65\}, \{0.25\}, \{0.25\} \rangle$	$\langle \{0.55, 0.65\}, \{0.45, 0.25\}, \{0.35, 0.25\} \rangle$
R_2	$\langle \{0.65\}, \{0.25\}, \{0.25\} \rangle$	$\langle \{0.55\}, \{0.45\}, \{0.35\} \rangle$	$\langle \{0.65\}, \{0.25\}, \{0.25\} \rangle$	$\langle \{0.65\}, \{0.25\}, \{0.25\} \rangle$
R_3	$\langle \{0.65\}, \{0.25\}, \{0.25\} \rangle$	$\langle \{0.55\}, \{0.45\}, \{0.35\} \rangle$	$\langle \{0.65\}, \{0.25\}, \{0.25\} \rangle$	$\langle \{0.65\}, \{0.25\}, \{0.25\} \rangle$
R_4	$\langle \{0.65\}, \{0.25\}, \{0.25\} \rangle$	$\langle \{0.55\}, \{0.45\}, \{0.35\} \rangle$	$\langle \{0.65\}, \{0.25\}, \{0.25\} \rangle$	$\langle \{0.55, 0.65\}, \{0.45, 0.25\}, \{0.35, 0.25\} \rangle$
R_5	$\langle \{0.55, 0.65\}, \{0.45, 0.25\}, \{0.35, 0.25\} \rangle$	$\langle \{0.55\}, \{0.45\}, \{0.35\} \rangle$	$\langle \{0.65\}, \{0.25\}, \{0.25\} \rangle$	$\langle \{0.65\}, \{0.25\}, \{0.25\} \rangle$
R_6	$\langle \{0.65\}, \{0.25\}, \{0.25\} \rangle$	$\langle \{0.55\}, \{0.45\}, \{0.35\} \rangle$	$\langle \{0.65\}, \{0.25\}, \{0.25\} \rangle$	$\langle \{0.55, 0.65\}, \{0.45, 0.25\}, \{0.35, 0.25\} \rangle$
R_7	$\langle \{0.65\}, \{0.25\}, \{0.25\} \rangle$	$\langle \{0.55\}, \{0.45\}, \{0.35\} \rangle$	$\langle \{0.65\}, \{0.25\}, \{0.25\} \rangle$	$\langle \{0.65\}, \{0.25\}, \{0.25\} \rangle$
R_8	$\langle \{0.65\}, \{0.25\}, \{0.25\} \rangle$	$\langle \{0.55\}, \{0.45\}, \{0.35\} \rangle$	$\langle \{0.65\}, \{0.25\}, \{0.25\} \rangle$	$\langle \{0.65\}, \{0.25\}, \{0.25\} \rangle$
R_9	$\langle \{0.65\}, \{0.25\}, \{0.25\} \rangle$	$\langle \{0.55\}, \{0.45\}, \{0.35\} \rangle$	$\langle \{0.65\}, \{0.25\}, \{0.25\} \rangle$	$\langle \{0.55, 0.65\}, \{0.45, 0.25\}, \{0.35, 0.25\} \rangle$
R_{10}	$\langle \{0.65\}, \{0.25\}, \{0.25\} \rangle$	$\langle \{0.55\}, \{0.45\}, \{0.35\} \rangle$	$\langle \{0.65\}, \{0.25\}, \{0.25\} \rangle$	$\langle \{0.65\}, \{0.25\}, \{0.25\} \rangle$
	$\langle P_5 \rangle$	$\langle P_6 \rangle$	$\langle P_7 \rangle$	$\langle P_8 \rangle$
R_1	$\langle \{0.55, 0.65\}, \{0.45, 0.25\}, \{0.35, 0.25\} \rangle$	$\langle \{0.65\}, \{0.25\}, \{0.25\} \rangle$	$\langle \{0.65\}, \{0.25\}, \{0.25\} \rangle$	$\langle \{0.65\}, \{0.25\}, \{0.25\} \rangle$
R_2	$\langle \{0.55, 0.65\}, \{0.45, 0.25\}, \{0.35, 0.25\} \rangle$	$\langle \{0.65\}, \{0.25\}, \{0.25\} \rangle$	$\langle \{0.65\}, \{0.25\}, \{0.25\} \rangle$	$\langle \{0.65\}, \{0.25\}, \{0.25\} \rangle$
R_3	$\langle \{0.55, 0.65\}, \{0.45, 0.25\}, \{0.35, 0.25\} \rangle$	$\langle \{0.65\}, \{0.25\}, \{0.25\} \rangle$	$\langle \{0.65\}, \{0.25\}, \{0.25\} \rangle$	$\langle \{0.65\}, \{0.25\}, \{0.25\} \rangle$
R_4	$\langle \{0.55, 0.65\}, \{0.45, 0.25\}, \{0.35, 0.25\} \rangle$	$\langle \{0.65\}, \{0.25\}, \{0.25\} \rangle$	$\langle \{0.65\}, \{0.25\}, \{0.25\} \rangle$	$\langle \{0.65\}, \{0.25\}, \{0.25\} \rangle$
R_5	$\langle \{0.55, 0.65\}, \{0.45, 0.25\}, \{0.35, 0.25\} \rangle$	$\langle \{0.65\}, \{0.25\}, \{0.25\} \rangle$	$\langle \{0.65\}, \{0.25\}, \{0.25\} \rangle$	$\langle \{0.65\}, \{0.25\}, \{0.25\} \rangle$
R_6	$\langle \{0.55, 0.65\}, \{0.45, 0.25\}, \{0.35, 0.25\} \rangle$	$\langle \{0.65\}, \{0.25\}, \{0.25\} \rangle$	$\langle \{0.65\}, \{0.25\}, \{0.25\} \rangle$	$\langle \{0.65\}, \{0.25\}, \{0.25\} \rangle$
R_7	$\langle \{0.55, 0.65\}, \{0.45, 0.25\}, \{0.35, 0.25\} \rangle$	$\langle \{0.65\}, \{0.25\}, \{0.25\} \rangle$	$\langle \{0.65\}, \{0.25\}, \{0.25\} \rangle$	$\langle \{0.65\}, \{0.25\}, \{0.25\} \rangle$
R_8	$\langle \{0.55, 0.65\}, \{0.45, 0.25\}, \{0.35, 0.25\} \rangle$	$\langle \{0.65\}, \{0.25\}, \{0.25\} \rangle$	$\langle \{0.65\}, \{0.25\}, \{0.25\} \rangle$	$\langle \{0.65\}, \{0.25\}, \{0.25\} \rangle$
R_9	$\langle \{0.55, 0.65\}, \{0.45, 0.25\}, \{0.35, 0.25\} \rangle$	$\langle \{0.65\}, \{0.25\}, \{0.25\} \rangle$	$\langle \{0.65\}, \{0.25\}, \{0.25\} \rangle$	$\langle \{0.65\}, \{0.25\}, \{0.25\} \rangle$
R_{10}	$\langle \{0.55, 0.65\}, \{0.45, 0.25\}, \{0.35, 0.25\} \rangle$	$\langle \{0.65\}, \{0.25\}, \{0.25\} \rangle$	$\langle \{0.65\}, \{0.25\}, \{0.25\} \rangle$	$\langle \{0.65\}, \{0.25\}, \{0.25\} \rangle$

approaches. The outcomes obtained by the compared methods are presented in Table 6.

Table 6 shows that the ranking of 8 policies according to Saqlain's method is $P_5 \succ P_6 \succ P_4 \succ P_1 \succ P_3 \succ P_7 \succ P_8 \succ P_2$ while Zulqarnain's method ranks them as $P_5 \succ P_8 \succ$

TABLE 4. Distance values to PIS and NIS of policies

Policies	d^+	d^-
P_1	0.40556	0.77132
P_2	0.31868	0.63240
P_3	0.42534	0.84251
P_4	0.36830	0.72069
P_5	0.33974	0.63940
P_6	0.38518	0.73729
P_7	0.36167	0.69369
P_8	0.33932	0.52766

TABLE 5. the correlation coefficient values and ranking of 8 policies

Policies	CC	Ranking
P_1	0.34460	3
P_2	0.33507	8
P_3	0.33548	7
P_4	0.33820	6
P_5	0.34698	2
P_6	0.34315	4
P_7	0.34270	5
P_8	0.39139	1

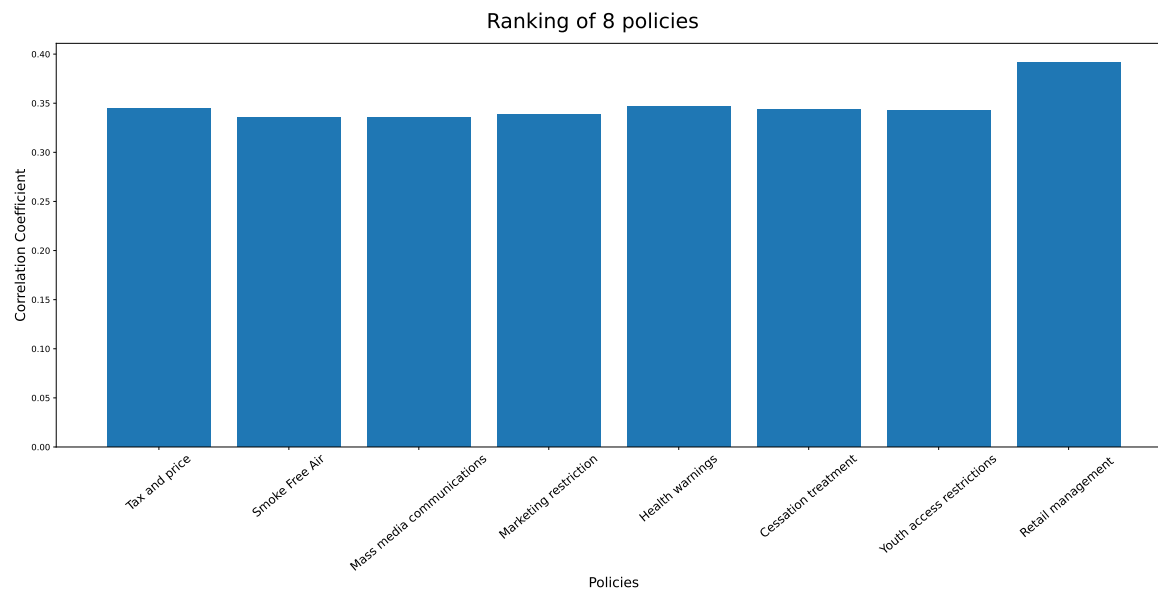


FIGURE 2. the correlation coefficient values

TABLE 6. Tangent similarity values and score values of 08 policies

Policies	Similarity value [29]	Ranking	Score value [36]	Ranking
P_1	0.95560	4	0.05136	3
P_2	0.94371	8	0.01725	7
P_3	0.95222	5	0.01037	8
P_4	0.95664	3	0.02553	6
P_5	0.96783	1	0.07087	1
P_6	0.95878	2	0.03369	4
P_7	0.94567	6	0.03181	5
P_8	0.94545	7	0.05295	2

$P_1 \succ P_6 \succ P_7 \succ P_4 \succ P_2 \succ P_3$. Both methodologies indicate that P_5 is the best policy for addressing cigarette use reduction problems. Meanwhile, the result of our model gives the ranking $P_8 \succ P_5 \succ P_1 \succ P_6 \succ P_7 \succ P_4 \succ P_3 \succ P_2$ identifying P_8 as the best policy. This result shows that the proposed model based on GNHSS theory, performs effectively in generalized neutrosophic hypersoft environments and has a strong impact on decision-making, particularly hesitancy occur during the estimation phase. The model also provides useful solutions to the critical and challenging of cigarette use reduction.

6. Conclusions

The ambiguity, uncertainty, and determination of data in practical terms make it increasingly challenging to solve the decision-making problem. It is crucial to consider the interconnections among sub-attributes and the availability of decision-makers. Traditional Neutrosophic Hypersoft set environments are inadequate for addressing such complexities. Hence, there arose an urgent necessity to establish a new methodology to resolve these challenges. In this work, we present a generalized neutrosophic hypersoft set based on the combination of NHSS theory and a hesitant fuzzy set. Subsequently, to complete the mathematical space of the proposed theory, the study also introduces basic operations and measures of GNHSS such as certainty, accuracy and aggregation operators. This study constructs the GNHSS-TOPSIS decision-making model to solve decision-making for related problems. Finally, the feasibility and soundness of the suggested model in addressing decision-making problems are demonstrated. This is illustrated with a real-world instance involving the solution of cigarette use reduction, a topic that has been a prominent societal concern over an extended period. However, this research is limited to a decision-making method based on GNHSS theory and its application to a single real-life case study. The future work will focus on developing and applying MCDM based on GNHSS theory to other real-life problems such as water treatment

and energy, etc. and integrating GNHSS theory to other domains, such as machine learning and data mining.

Funding: This research is funded by Thuongmai University, Hanoi, Vietnam

Acknowledgments: The corresponding author would like to thank Professor Florentin Smarandache for valuable comments and suggestions.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Abbas, S., Alnoor, A., Yin, T. S., Sadaa, A. M., Muhsen, Y. R., Khaw, K. W., & Ganesan, Y. (2023). Antecedents of trustworthiness of social commerce platforms: A case of rural communities using multi group sem & mcdm methods. *Electronic commerce research and applications*, 62, 101322.
2. Yu, K., Wu, Q., Chen, X., Wang, W., & Mardani, A. (2023). An integrated mcdm framework for evaluating the environmental, social, and governance (esg) sustainable business performance. *Annals of Operations Research*, 1–32.
3. Stanković, J. J., Marjanović, I., Papathanasiou, J., & Drezgić, S. (2021). Social, economic and environmental sustainability of port regions: Mcdm approach in composite index creation. *Journal of Marine Science and Engineering*, 9(1), 74.
4. Luczak, A., & Just, M. (2020). A complex mcdm procedure for the assessment of economic development of units at different government levels. *Mathematics*, 8(7), 1067.
5. Yu, X., Wu, X., & Huo, T. (2020). Combine mcdm methods and pso to evaluate economic benefits of high-tech zones in china. *Sustainability*, 12(18), 7833.
6. Yilmaz, B. Ö., Tozan, H., & Karadayı, M. A. (2020). Multi-criteria decision making (mcdm) applications in military healthcare field. *Journal of Health Systems and Policies*, 2(2), 149–181.
7. Stević, Ž., Pamučar, D., Puška, A., & Chatterjee, P. (2020). Sustainable supplier selection in healthcare industries using a new mcdm method: Measurement of alternatives and ranking according to compromise solution (marcos). *Computers & industrial engineering*, 140, 106231.
8. Zadeh, L. A. (1965). Fuzzy sets. *Information and control*, 8(3), 338–353.
9. Atanassov, K. T., & Atanassov, K. T. (1999). *Intuitionistic fuzzy sets*. Springer.
10. Torra, V. (2010). Hesitant fuzzy sets. *International journal of intelligent systems*, 25(6), 529–539.
11. Thong, N. T., Smarandache, F., Hoa, N. D., Son, L. H., Lan, L. T. H., Giap, C. N., Son, D. T., & Long, H. V. (2020). A novel dynamic multi-criteria decision making method based on generalized dynamic interval-valued neutrosophic set. *Symmetry*, 12(4), 618.

12. Ahmad, F., Adhami, A. Y., & Smarandache, F. (2018). Single valued neutrosophic hesitant fuzzy computational algorithm for multiobjective nonlinear optimization problem. *Neutrosophic sets and systems*, 22, 76–86.
13. Ye, J. (2015). Multiple-attribute decision-making method under a single-valued neutrosophic hesitant fuzzy environment. *Journal of Intelligent Systems*, 24(1), 23–36.
14. Jeon, J., Suvitha, K., Arshad, N. I., Kalaiselvan, S., Narayanamoorthy, S., Ferrara, M., & Ahmadian, A. (2023). A probabilistic hesitant fuzzy mcdm approach to evaluate india's intervention strategies against the covid-19 pandemic. *Socio-Economic Planning Sciences*, 89, 101711.
15. Xian, S., Ma, D., & Feng, X. (2024). Z hesitant fuzzy linguistic term set and their applications to multi-criteria decision making problems. *Expert Systems with Applications*, 238, 121786.
16. Smarandache, F. (1999). A unifying field in logics: Neutrosophic logic. In *Philosophy* (pp. 1–141). American Research Press.
17. Fan, C., Han, M., & Fan, E. (2024). A pythagorean language neutrosophic set method for the evaluation of water pollution control technology in pulp and paper industry. *Engineering Applications of Artificial Intelligence*, 133, 108032.
18. Farid, H. M. A., & Riaz, M. (2023). Single-valued neutrosophic dynamic aggregation information with time sequence preference for iot technology in supply chain management. *Engineering Applications of Artificial Intelligence*, 126, 106940.
19. Lan, L. T. H., Thong, N. T., Smarandache, F., Giang, N. L., et al. (2023). An anp-topsis model for tourist destination choice problems under temporal neutrosophic environment. *Applied Soft Computing*, 136, 110146.
20. Molodtsov, D. (1999). Soft set theory—first results. *Computers & mathematics with applications*, 37(4-5), 19–31.
21. Smarandache, F. (2018). Extension of soft set to hypersoft set, and then to plithogenic hypersoft set. *Neutrosophic sets and systems*, 22(1), 168–170.
22. Saeed, M., Siddique, M. K., Ahsan, M., Ahmad, M. R., & Rahman, A. U. (2021). A novel approach to the rudiments of hypersoft graphs. *Theory and Application of Hypersoft Set*, Pons Publication House, Brussel, 203–214.
23. Debnath, S., & Kamacı, H. (2023). Hypersoft game theory models and their applications in multi-criteria decision making. *Pamukkale Üniversitesi Mühendislik Bilimleri Dergisi*, 29(7), 680–691.
24. Saqlain, M., Moin, S., Jafar, M. N., Saeed, M., & Smarandache, F. (2020). *Aggregate operators of neutrosophic hypersoft set*. Infinite Study.
25. Saeed, M., Wahab, A., Ali, M., Ali, J., & Bonyah, E. (2023). An innovative approach to passport quality assessment based on the possibility q-rung ortho-pair fuzzy hypersoft set. *Heliyon*, 9(9).

26. Zulqarnain, R. M., Xin, X. L., & Saeed, M. (2021). A development of pythagorean fuzzy hypersoft set with basic operations and decision-making approach based on the correlation coefficient. *Theory Appl. Hypersoft Set*, 2021, 85–106.
27. Saqlain, M., Riaz, M., Saleem, M. A., & Yang, M.-S. (2021). Distance and similarity measures for neutrosophic hypersoft set (nhss) with construction of nhss-topsis and applications. *Ieee Access*, 9, 30803–30816.
28. Saqlain, M., Saeed, M., Ahmad, M. R., & Smarandache, F. (2019). *Generalization of topsis for neutrosophic hypersoft set using accuracy function and its application*. Infinite Study.
29. Saqlain, M., Jafar, N., Moin, S., Saeed, M., & Broumi, S. (2020). Single and multi-valued neutrosophic hypersoft set and tangent similarity measure of single valued neutrosophic hypersoft sets. *Neutrosophic Sets and Systems*, 32(1), 317–329.
30. Saqlain, M., Garg, H., Kumam, P., & Kumam, W. (2023). Uncertainty and decision-making with multi-polar interval-valued neutrosophic hypersoft set: A distance, similarity measure and machine learning approach. *Alexandria Engineering Journal*, 84, 323–332.
31. Jafar, M. N., Saeed, M., Khan, K. M., Alamri, F. S., & Khalifa, H. A. E.-W. (2022). Distance and similarity measures using max-min operators of neutrosophic hypersoft sets with application in site selection for solid waste management systems. *Ieee Access*, 10, 11220–11235.
32. Jafar, M. N., Saeed, M., Saqlain, M., & Yang, M.-S. (2021). Trigonometric similarity measures for neutrosophic hypersoft sets with application to renewable energy source selection. *Ieee Access*, 9, 129178–129187.
33. Rahman, A. U., Saeed, M., Mohammed, M. A., Abdulkareem, K. H., Nedoma, J., & Martinek, R. (2023). Fppsv-nhss: Fuzzy parameterized possibility single valued neutrosophic hypersoft set to site selection for solid waste management. *Applied soft computing*, 140, 110273.
34. Dhumras, H., & Bajaj, R. K. (2023). On novel hellinger divergence measure of neutrosophic hypersoft sets in symptomatic detection of covid-19. *Neutrosophic Sets and Systems*, 55(1), 16.
35. Zhao, J., Li, B., Rahman, A. U., & Saeed, M. (2023). An intelligent multiple-criteria decision-making approach based on sv-neutrosophic hypersoft set with possibility degree setting for investment selection. *Management Decision*, 61(2), 472–485.
36. Zulqarnain, R. M., Ma, W. X., Siddique, I., Gurmani, S. H., Jarad, F., & Ahamad, M. I. (2023). Extension of aggregation operators to site selection for solid waste management under neutrosophic hypersoft set. *AIMS Mathematics*, 8(2), 4168–4201.
37. Ihsan, M., Saeed, M., Alanzi, A. M., & Khalifa, H. E.-W. (2023). An algorithmic multiple attribute decision-making method for heart problem analysis under neutrosophic hypersoft expert set with fuzzy parameterized degree-based setting. *PeerJ Computer Science*, 9, e1607.

38. Angelica, V. B. G., Favio, V. B. E., Manuel, V. M. V., Amandiz, C. V. D., & del Carmen, P. A. I. (2024). Model of tax culture impact on the financial sustainability of small and medium enterprises in ecuador based on neutrosophic hypersoft sets. *Neutrosophic Sets and Systems*, 69, 86–93.
39. Ajabnoor, N. (2024). Evaluation of distributed leadership in education using neutrosophic hypersoft set. *Neutrosophic Sets and Systems*, 72(1), 17.
40. Saeed, M., Ashraf, M., Ijaz, A., & Jaffar, M. N. (2024). Development of pythagorean neutrosophic hypersoft set with application in work life balance problem. *HyperSoft Set Methods in Engineering*, 1, 59–70.
41. Rahman, A. U., Saeed, M., Smarandache, F., & Ahmad, M. R. (2020). *Development of hybrids of hypersoft set with complex fuzzy set, complex intuitionistic fuzzy set and complex neutrosophic set*. Infinite Study.

Received: Dec. 26, 2024. Accepted: July 7, 2025